Abstract

In the previous paper [3] we established the maximum principles for the finite element solutions of the partial differential equation: $\Delta u - qu = f$ on a compact bordered Riemann surface $\overline{\Omega}$. In the present paper we shall improve and extend the results in the paper [3]. First we construct a triangulation $K$ of $\overline{\Omega}$ with width $h$ and introduce a class $S = S(K)$ of element functions on $K$. For a partition to two parts $C_1$ and $C_2$ of the boundary $\partial \Omega$, we define the finite element approximation $\omega_h \in S$ of the boundary value problem: $\Delta u - qu = f$ on $\Omega$, $u = \chi$ on $C_1$ and $\ast du = 0$ along $C_2$, where by $\ast du$ we denote the conjugate differential of $du$. We assume that all angles of 2-simplices of $K$ are $\leq \pi/2$. Under the assumption weaker than one in the paper [3], we shall exhibit that the inequality

$$|\omega_h| \leq \exp \left( \frac{4\pi M}{\sin \theta} \max_q q \left( \max_{c_1} |\chi| + \frac{2}{\sin \theta} \iint_{c_2} |f| \, dx \, dy \right) \right)$$

holds for sufficiently small $h$, where $\theta$ is the smallest value of all angles of 2-simplices of $K$ and $M$ is a constant. The last inequality will be very useful to obtain error estimates of the finite element solutions.